# 8 Probability

M11CH8

## 8.1. INTRODUCTION

The words probable, likely, possibly and chance are common in our everyday conversation. for example, we say "Most probably I will get grade A". "Probably it will rain today". "It is possible that price of fuel will rise" . "The chances of liberia winning the African cup are more". In each of the above statements there is a degree of uncertainty. This degree of uncertainty can be measured numerically with the help of "probability".

The theory of probability has its origin from:the game of chance, for instance, tossing of coins, throwing of dice, drawing a card from a pack of cards, lottery etc.

Today probability is an important branch of Mathematics with applications in statistics, economics, Industry, Science and Technology.

## 8.2. DEFINITION OF TERMS

Let us introduce certain terms used in the theory of probability.

### Experiment

It is an operation which produces some well-defined outcomes.
The following are example of an experiment.

1. Tossing a coin
2. Throwing of a die
3. Drawing a card from a well-shuffled pack of 52 cards.

In all the above examples of experiment, the outcome is not certain.

**Trial:** A trial is an action which produces one or more outcomes. In fact trial implies performing an experiment.

## Outcome

The result of an experiment is called outcome.

## Equally likely outcomes

In an experiment, the outcomes which have equal chance of occurring are called equally likely outcomes. For example, if we toss a coin, outcomes Head or Tail are equally likely. Also, if we throw a die, the outcomes, 1, 2, 3, 4, 5, 6 are equally likely. This is so because each number has the same possibility of showing up.

## Random sampling

This is choosing a sample from a papulation without being bias.

## Sample Space

The sample space of an experiment is the set of all possible outcomes of a trial of, the experiment. It is denoted by the letter S.

## Sample Space for Simple Experiments

Simple experiments involve performing an experiment once. For example, throwing a die once or tossing a coin once.

Let us now find the sample space of some simple experiments.

1. When we toss a fair coin once, either a Head (H) or Tail (T) will turn up.

   Therefore the set of possible outcomes (sample space), S = {H, T)

2. When we throw an unbiased die, the. outcomes, 1 or 2 or 3 or 4 or 5 or 6 are equally likely. Therefore the set of possible outcomes, S = { 1, 2, 3, 4, 5, 6}.

3. The sample space for drawing a card from a pack of cards bearing numbers 1 to 52 is S = {1 , 2, 3, 4, 5 .... 51, 52). Think about other simple experiments and write their sample space.

## Sample Space for Compound Experiments

Compound experiments involve the performing the same experiment more than once or performing two or more independent experiments at the same time. For example,the throw of a die twice or the toss of a coin, twice is a compound experiment.

Let us now find the sample space of some compound experiments.

1. When two fair coins are tossed

   S = {HH, HT, TH, TT}

2. When three fair coins are tossed

   {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

3. When two unbiased dice are thrown

   | | | | | | |
   |---|---|---|---|---|---|
   | 1,1 | 1,2 | 1,3 | 1,4 | 1,5 | 1,6 |
   | 2,1 | 2,2 | 2,3 | 2,4 | 2,5 | 2,6 |
   | 3,1 | 3,2 | 3,3 | 3,4 | 3,5 | 3,6 |
   | 4,1 | 4,2 | 4,3 | 4,4 | 4,5 | 4,6 |
   | 5,1 | 5,2 | 5,3 | 5,4 | 5,5 | 5,6 |
   | 6,1 | 6,2 | 6,3 | 6,4 | 6,5 | 6,6 |

   Note that the sample space for a die thrown twice is the same as two dice thrown simultaneously.

4. When a fair coin and an unbiased die are thrown.

   | | | | | | |
   |---|---|---|---|---|---|
   | H 1 | H 2 | H 3 | H 4 | H 5 | H 6 |
   | T 1 | T 2 | T 3 | T 4 | T 5 | 4 6 |

## 8.3. DEFINITION OF PROBABILITY

In this Chapter, we propose to study theoretical probability that attempts to predict what will happen. The basic difference between these two probabilities is that whilst, experimental probability is based on what has actually happened, theoretical probability attempts to predict what will happen.

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total number of all possible outcomes of the experiments}}$$

Thus, if the number of outcomes favourable to the event E is 'm' out of total 'n' equally likely outcomes,

then,                                   $P(E) = \dfrac{m}{n}.$

**Note:** Since m and n are integers, by the above definition, P(E) is always a rational number.

The above definition holds on the assumption that the outcomes of the experiment are equally likely.

## Limitations of Theoretical Probability

This definition is not true in the following cases:

1. If the events are not equally likely.
2. If the possible outcomes are infinite.
3. When the probability is not a rational number.

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}.$$

Also the probability that a number chosen is greater than 2 is $\frac{4}{6} = \frac{2}{3}$.

The probability that a number chosen is divisible by 3 is $\frac{2}{6} = \frac{1}{3}$, since, there are 2 numbers which are divisible by 3 out of a total of 6 numbers.

Similarly, in a class of 24 boys and 36 girls, the probability that a student selected from the class is a boy is $P(B) = \frac{24}{60} = \frac{2}{5}$, since, the number of boys in the class is 24 and the total number of students in the class is n(S) = 60. Also, the probability that a student chosen at random from the class is a girl is $P(G) = \frac{36}{60} = \frac{3}{5}$, since, there are 36 girls out of a total of 60 students.

## Notation

If E is an event, then $P(\bar{E})$ is the probability that E will occur. $\bar{E}$ is the event that E will not occur and so $P(\bar{E})$ is the probability that E will not occur. The sample space, S, is used to denote the set of all possible outcomes.
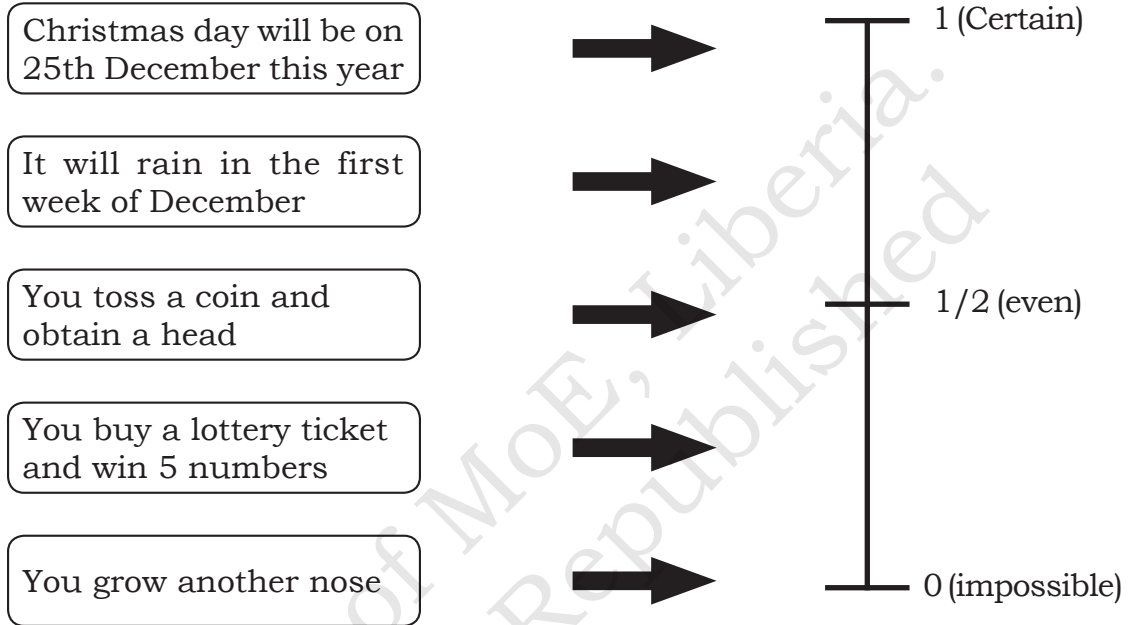
## Describing probabilities

We often make judgments as to whether an event will take place, and use words to describe how probable that event is. For example, we might say that it is likely that the sun will come up tomorrow, or that it is impossible to find somebody who is more than 3 m tall.

## The probability scale

Mathematics uses numbers to describe probabilities. Probabilities can be written as fractions, decimals or percentages. You can also use a **probability scale,** starting at 0 (impossible) and ending at 1 (certain).

If it is **certain** or **likely** that an event will occur then the probability is 1. If it is impossible or unlikely that the event will occur, then the probability is 0. If the outcomes have equal chance of occurring, then they are called equally likely outcomes.

Here are some events placed on the probability scale.

| | | |
|---|---|---|
| Christmas day will be on 25th December this year | → | 1 (Certain) |
| It will rain in the first week of December | → | |
| You toss a coin and obtain a head | → | 1/2 (even) |
| You buy a lottery ticket and win 5 numbers | → | |
| You grow another nose | → | 0 (impossible) |

## 8.4. LAWS OF PROBABILITY

1. The probability that an event E will occur lies between 0 and 1 inclusive i.e.

$$0 \leq P(E) \leq 1.$$

**Proof:** Since, E is a subset set of S.

i.e., $\qquad E \subseteq S \Rightarrow 0 \leq n(E) \leq n(S)$

Dividing the inequality by n(S), we have

$$\frac{0}{n(S)} \leq \frac{n(E)}{n(S)} \leq \frac{n(S)}{n(S)}$$

∴ $\qquad 0 \leq P(E) \leq 1$

We can say from above that the probability of any event lies between 0 and 1 inclusive

2. The probability that the sample space will occur is 1. i.e. $P(S) = 1$

**Proof:** $P(S) = \dfrac{n(S)}{n(S)} = 1$

Thus, the probability of a certain event is 1.

3.  P(∅)= 0 where ∅ is the null set.

    **Proof:** $P(\phi) = \dfrac{n(\phi)}{n(S)} = \dfrac{0}{n(S)} = 0$

    Thus the probability of an impossible event is 0.

**4.** If the probability that an event E will occur denoted by P(E)= r, then the probability that the event will not occur denoted by

$P(\overline{E}) = 1 - P(E) = 1 - r$

**Proof:** The complement $\overline{E}$ of E contains those sample points of S which are not in E.

$\therefore n(\overline{E}) = n(s) - n(E)$

Dividing both sides by n(S), we have

$$\frac{n(\overline{E})}{n(S)} = \frac{n(S)}{n(S)} - \frac{n(E)}{n(S)}$$

$\therefore \qquad\qquad\qquad P(\overline{E}) = 1 - P(E)$

For example, if the probability that it will rain on a particular day is $\dfrac{1}{4}$, then the probability that it will not rain on that day is

$$1 - \frac{1}{4} = \frac{3}{4}$$

Again, if the probability that it will rain tomorrow is 1 – 0.21 = 0.79.

## Illustrative examples

These types of examples are **simple cases** that involve the use of the definition of probability. The definition of an even number, **prime** number, **odd** number and multiples of a number may also be helpful.

**Example 1:** *A number is chosen at random from the integers  5 to 25 inclusive. Find the probability that the number is a*

(i)  *multiple of 3*

(ii)  *multiple of 5*

**Solution:** The set of possible outcomes

S = {5, 6, 7, 8, 9, 10, 11, ...., 22, 23, 24, 25}

$\Rightarrow$                                          n(S) = 21

E = {multiples of 3} = {6, 9, 12, 15, 18, 21, 24}

$\Rightarrow$                                          n(E) = 7

F = {multiples of 5} = {5, 10, 15, 20, 25}

$\Rightarrow$                                          n(F) = 5

  (i)  P(number is a multiple of 3) = $\dfrac{7}{21}$

 (ii)  P(number is a multiple of 5) = $\dfrac{5}{21}$

**Example 2:** *A number is chosen at random from the set*

*S = {4, 7, 10, 13, 16, 19}.*

*What is the probability that the number is*

  (i)  *even*             (ii)   *odd*

(iii)  *greater than 10*   (iv)   *less than 10*

  (v)  *between 4 and 19*

**Solution:** The set of possible outcomes

S = {4, 7, 10, 13, 16, 19} $\Rightarrow$ n(S) = 6

  (i)  P(number is even) = $\dfrac{3}{6} = \dfrac{1}{2}$

 (ii)  P(number is odd) = $\dfrac{3}{6} = \dfrac{1}{2}$

(iii)  P(number is greater than 10) = $\dfrac{3}{6} = \dfrac{1}{2}$

(iv)  P(number is less than 10) = $\dfrac{2}{6} = \dfrac{1}{3}$

  (v)  P(number is between 4 and 19) = $\dfrac{4}{6} = \dfrac{2}{3}$

**Example 3:** *In a class of 20 boys and 35 girls What is the probability that a student selected at random is a boy.*

**Solution:** Total number of students in the class 20 + 35 = 55;

Number of boys in the class = 20

$$\therefore \quad P(\text{a boy is selected}) = \frac{20}{55} = \frac{4}{11}$$

## Relative frequency

Probabilities can also be estimated from the results or outcomes of an experiment. The **relative frequency** of an event is defined as:

$$\frac{Frequency\ of\ the\ event}{Total\ frequency}$$

The relative frequency gives an estimate of the probability of an event. When the total frequency is very large the relative frequency provides a good estimates.

**Example 4:** *In a survey of 150 families with four 'children to find the number of girls the following results were obtained.*

| Number of girls | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Frquency | 8 | 41 | 55 | 33 | 13 |

*Estimate the probability that a family with four children will have*

   (i) *two girls*       (ii) *no girls*

 (iii) *one boy*        (iv) *four boys*

   (v) *at least three girls*

## Solution:

   (i) $P(\text{two girls}) = \dfrac{55}{150} = \dfrac{11}{30}$

  (ii) $P(\text{no girls}) = \dfrac{8}{150} = \dfrac{4}{75}$

 (iii) $P(\text{one boy}) = P(\text{three girls}) = \dfrac{33}{150} = \dfrac{11}{50}$

 (iv) $P(\text{four boys}) = P(\text{no girls}) = \dfrac{8}{150} = \dfrac{4}{75}$

  (v) $P(\text{at least three girls })$

      $= P(\text{three or more girls})$

$$= \frac{33+13}{150} = \frac{46}{150} = \frac{23}{75}$$

## 8.5. EVENT

**Event:** An event for an experimental is the collection of favourable outcomes of the experiment.

It is generally denoted by E or A, B, etc.

**For example:**

  (i)  In tossing a coin, if the appearance of head occurs as the event, then E = the event of **appearing head**.

 (ii)  In throwing a die, if the appearance of the even number occurs as the event, then E = the event of appearing even number.

      In Simple words, an event is simply an outcome of an experiment.

### Types of events

### 1. Certain events

An event which contains all the sample space is a certain event.

### 2. Impossible event .

An event which does-not contain any sample point is an impossible event.

### 3. Complement of an event

Let S be the sample space and A be an event of S. The set of all those sample points which are in S but not in A is called the complement of the event A and is denoted by $\overline{A}$. See fig. 8.1. Note that the complement of a certain event is an impossible event.
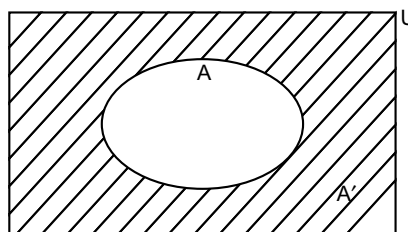


**Fig. 8.1.**

## 4. Compound events

Events can be combined by the words '**or**' and '**and**'. Events which are thus combined are called compound events. The words 'or' and 'and' correspond to ∪ and '∩' respectively in sets. Let A and B be two events. The compound events:

1. A ∪ B (i.e., A or B) means either event A or event B or both events occur.
2. A ∩ B (i.e. A and B) means the event A and B occur together.

## 5. Mutually exclusive events

If A and B are events such that the two events cannot occur together or at the same time, then we say that the events A and B are **mutually exclusive** events. Thus if A ∩ B = ∅, then events A and B are mutually exclusive (fig 8.2). For example, , when you toss a coin, you can get either a head or a tail, not both at the same time. So the events of getting a head and getting a tail are mutually exclusive.

In this case, A and B are disjoint sets.
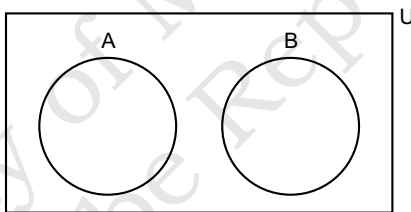


**Fig. 8.2.**

## 6. Independent events

Two events A and B, are said to **independent** if the occurrence of A does not affect the occurrence of B and vice versa. For example, if a fair die is thrown twice; let the event A = throwing a six on the first throw and B = throwing a six on the second throw. The events A and B have no, influence over each other and are therefore said to be independent.

We can use our set language for union, intersection, and complement to describe events .

## 7. Union of Two Events

If E and F are two events; then E ∪ F ts the union of the two events and consists of the set of outcomes that are in E or F.

Thus, the event E ∪ F is the event that "E or F occur." Refer to

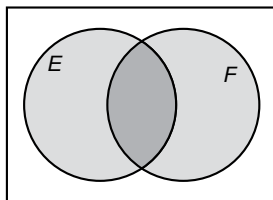Figure 8.3 where the event E $\cup$ F is the shaded region on the Venn diagram.



**Fig. 8.3.**

## 8. Intersection, of Two Events

If E and F are two events, then E $\cap$ F is the intersection of the two events and consists of the set of outcomes that are in both E and F.

Thus, the event E $\cap$ F is the event that "E and F both occur." Refer to Figure 8.3 where the event E $\cap$ F is the region where E and F overlap.

## 9. Complement of an Event

If E is an event, then $E^c$ is the Complement of E and consists of the set of outcomes that are-not E.

Thus, the event $E^c$ is the event that "E does not occur."

**Example 5**: *Determining Union, Intersection, and Complement Consider the sample space given in Example 2. Let E consist of those outcomes for which the number of dots on the top faces of both dice is 2 or 4. Let F be the event that the sum of the number of dots on the top faces of the two dice is 6. Let G be the event that the sum of the number of dots on the top faces of the two dice is less than 11.*

    *a. List the elements of E and F.*

    *b. Find E $\cup$ F.*

    *c. Find E $\cap$ F.*

    *d. Find $G^c$.*

**Solution:**

    a. E = {(2, 2), (2, 4), ( 4, 2), ( 4, 4)} and E = {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)}

    b. E $\cup$ F = {(2,2), (2,4), (4,2) , (4,4) , (1, 5), (3,3), (5, 1)}

    c. E $\cap$ F = {(2, 4), (4, 2)}

    d. $G^c$ = {(5, 6), (6, 5), (6, 6)}

If S is a sample space, $\varnothing \subseteq S$, and thus $\varnothing$ is an event. We call the event 0 the impossible event since the event $\varnothing$ means that no outcome has occurred, whereas, in any experiment some outcome must occur.

## ▌ 8.6. BASIC CONCEPTS OF SETS

We begin here with some definitions of the language and notation used when working with sets. The most basic definition is "What is a set?" A set is a collection of items. These items are referred to as the elements or members of the set. For example, the set containing the numbers 1, 2, and 3 would be written {1, 2, 3}. Notice that the set is contained in curly brackets. This will help us distinguish sets from other mathematical objects.

When all the elements of the set are written out, we refer to this as roster notation. So the set containing the first 10 letters in the English alphabet would be written as {a, b, c, d, e, f, g, h, i, j} in roster notation. If we wanted to refer to this set without writing all the elements, we could define the set in terms of its, properties. This is called set-builder notation. So we write

{x|x is one of the first 10 letters in the English alphabet}

This is read "the set of all x such that x is one of the first 10 letters in the English, alphabet". If we will be using a set more than once in a discussion, it is useful to define the set with a symbol, usually an uppercase letter. So

S = {a, b, c, d, e, f, g, h, i, j}

We can say c is an element of the set {a, b, c, d, e, f, g, h, i, j} or simply write $c \in S$. The symbol $\in$ is read "is an element of". We can also say that the set R = {c} is a subset of our larger set S as every element in the set R is also in the set S.

### Subsets

If every element of a set A is also an element another set B, we say that A is a subset of B and write $A \subseteq B$; If A is not a subset of B, we write

$$A \nsubseteq B.$$

Thus {1, 2, 4} $\subseteq$ {1, 2, 3, 4}, but {1, 2, 3, 4} $\nsubseteq$ {1 ,2,4}. Since, every element in A is in A, we can write $A \subseteq A$. If there is a set B and every

element in the set B is also in the set A but B ≠ A, we say that B is a proper subset of A. This is written, as B ⊂ A. Note the proper subset symbol ⊂ is lacking the small horizontal line that the subset symbol ⊆ has. The difference is rather like the difference between, < and ≤.

Some sets have no elements at all. We need some notation for this, simply leaving a blank space will not do!

## Empty Set

The empty set, written as φ or { }, is the set with no elements.

The empty set can be used to conveniently indicate that an equation has no solution.

## For example:

$$\{x | x \text{ is real and, } x^2 = -1\} = \phi$$

By the definition of subset, given any set A, we must have $\phi \subseteq A$.

**Example 6:** *Finding Subsets Find all the subsets of {a, b, c}.*

**Solution:** The subsets are

$$\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$$

The empty set is the set with no elements. At the other extreme is the **universal set**. This set is the set of all elements being considered and is denoted by U. If, for example, we are to take a national survey of voter satisfaction with the president, the universal set is the set of all voters in this country. If the survey is to determine the effects of smoking on pregnant women, the universal set is the set of all pregnant women. The context of the problem under discussion will determine the universal set for that problem. The universal set must contain every element under discussion.

A **Venn diagram** is a way of visualizing sets. The universal set is represented by a rectangle and sets are represented as circles inside the universal set. For example, given a universal set U and a set A, Figure 8.4 is a Venn diagram that visualizes the concept that A ⊂ U. Figure 8.4 also visualizes the concept B ⊂ A. The U above the rectangle will be dropped in later diagrams as we will abide by the convention that the rectangle always represents the universal set.
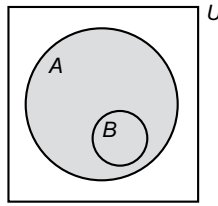
**Fig. 8.4.**

## Set Operations

The first set operation we consider is the complement. The complement of set A are those members of set U that do not belong to A.

## Complement

Given a universal set U and a set $A \subset U$, the complement A, written $A^c$ is the set of all elements that are in U but not in A, that is,

$$A^c = \{x | x \in U, x \notin A\}$$

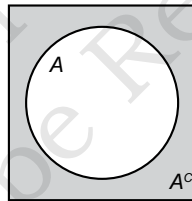A Venn diagram visualizing $A^c$ is shown in Figure 8.5. Some alternate notations for the complement of a set are $A^1$ and $\bar{A}$.



**Fig. 8.5. $A^c$ is shaded.**

**Example 7:** *The Complements of Sets Let U = {l, 2, 3, 4, 5, 6, 7, 8, 9} A = {1, 3, 5, 7, 9}, B = {1, 2, 3, 4, 5}. Find $A^c$, $B^c$, $U^c$, $\phi^c$, and $(A^c)^c$ in roster notation.*

**Solution:** We have

$$A^c = \{2, 4, 6, 8\}$$
$$B^c = \{6, 7, 8, 9\}$$
$$U^c = \phi$$
$$\phi^c = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} = U$$
$$(A^c)^c = \{2, 4, 6, 8\}^c$$
$$= \{l, 3, 5, 7, 9\} = A$$

Note that in the example above we found $U^c = \varnothing$ and $\varnothing^c = U$. Additionally $(A^c) = A$. This can be seen using the Venn diagram in Figure 8.5, since the complement of $A^c$ is all elements in U but not in $A^c$ which is the set A. These three rules are called the Complement Rules.

Complement Rules

If U is a universal set, we must always have

$$U^c = \phi, \phi^c = U$$

If A is any subset of a universal set U, then

$$(A^c)^c = A$$

The next set operation is the union of two sets. This set includes the members of both sets A and B. That is, if an element belongs to set A or set B then it belong, to the union of A and B.

## Set Union

The union of two sets A and B, written $A \cup B$, is the set of all elements that belong to A, or to B, or to both. Thus

$$A \cup B = \{x | x \in A \text{ or } x \in B \text{ or both}\}$$

$$A \cup B = \{x | x \in A \text{ or } x \in B\}$$

**Example 8:** *The Union of Two Sets Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2, 3, 4\}$ and $B = \{1, 4, .5, 6\}$. Find $A \cup B$ and $A \cup Ac$.*

**Solution:** We begin with the first set and join to it any elements in the second

set that are not already there. Thus

$$A \cup B = \{1, 2, 3, 4\} \cup \{1, 4, 5, 6\}$$
$$= \{1, 2, 3, 4, 5, 6\}$$

Since $A^c = \{5, 6\}$ we have

$$A \cup A^C = \{1, 2, 3, 4\} \cup \{5, 6\}$$
$$= \{1, 2, 3, 4, 5, 6\} = U$$

The second result, $A \cup A^c = U$ is generally true. From Figure 8.5, we can see that if U is a universal set and $A \subset U$, then
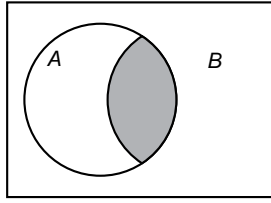
$$A \cup A^6 = U$$

**Fig. 8.6.**

## Set Intersection

The intersection of two sets A and B, written $A \cap B$, is the set of all elements that belong to both the set A and to the set B. Thus

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

A Venn diagram is shown in Figure 8.6 with the intersection shaded.

**Example 9:** *The Intersection of Two Sets Find*

    a.  *{a, b, c, d} ∩ {a, c, e}*

    b.  *{a, b} ∩ {c, d}*

## Solution:

    a.   Only a and c are elements of both of the sets. Thus

$$\{a, b, c, d\} \cap \{a, c, e\} = \{a, c\}$$

    b.   The two sets {a, b} and {c,d} have no elements in common. Thus

$$\{a, b\} \cap \{c, d\} = \varnothing$$

## Disjoint Sets

The sets {a, b} and {c, d} have no elements in common. These sets are called disjoint and can be visualized in Figure 8.7.



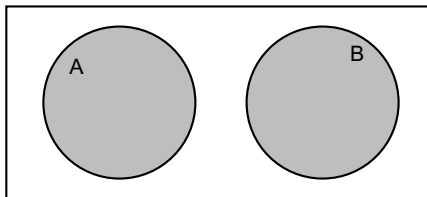**Fig. 8.7. *A* and *B* are disjoint.**

Two sets A and B are disjoint if they have no elements in common, that is, if $A \cap B = \varnothing$.

An examination of Figure 8.7 or referring to the definition of $A^c$ indicates that for any set A, A and $A^c$ are disjoint. That is,

$$A \cap A^c = \varnothing$$

## Additional Laws for Sets

There are a number of laws for sets. They are referred to as commutative, associative, distributive, and De Morgan laws. We will consider two of these laws in the following examples

**Example 10:** *Establishing a De Morgan Law Use a Venn diagram to show that*

$$(A \cup B)^C = A^c \cap B^c$$

**Solution:** We first consider the right side of this equation. Figure 8.8 shows Venn diagram of $A^c$ and $B^c$ and $A^c \cap B^c$. We then notice from Figure 8.5 that this is $(A \cup B)^c$
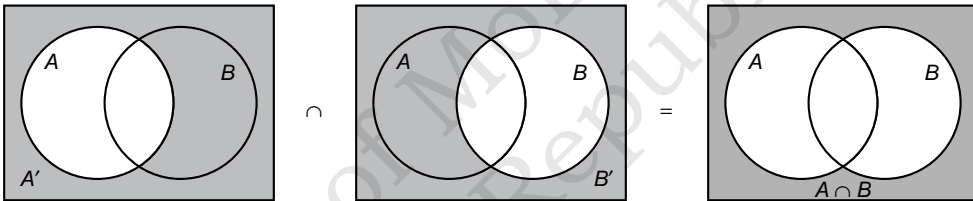


**Fig. 8.8.**

**Example 11:** *Establishing the Distributive Law for Union Use a Venn diagram to show that*

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

**Solution:** Consider first the left side of this equation. In Figure 8.9a the sets A, $B \cap C$, and the union of these two are shown. Now for the right side of the, equation refer to Figure 8.9b, where the sets $A \cup B$, $A \cup C$, and the intersection of these two sets are shown. We have the same set in both cases.
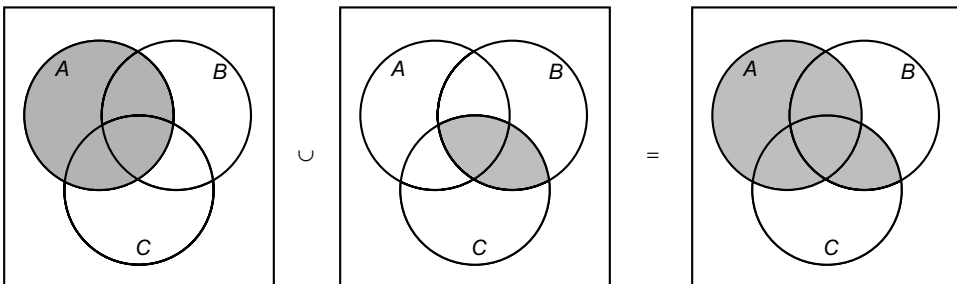


**Fig. 8.9. a**

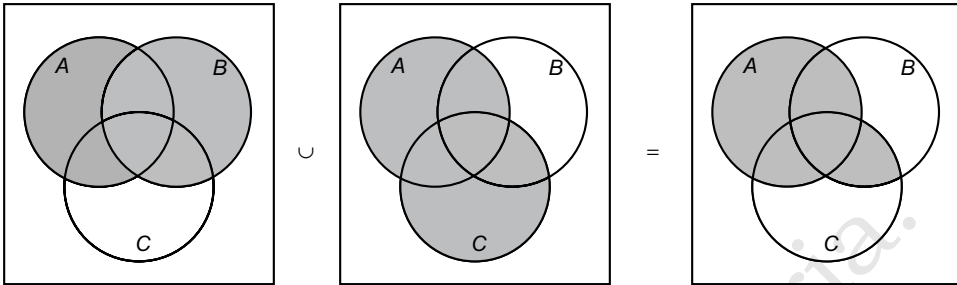We can summarize the laws we have found in the following list.



**Fig. 8.9. b**

We can summarize the laws we have found in the following list.

## Laws for Set Operations

| | |
|---|---|
| $A \cup B = B \cup A$ | Commutative law for union |
| $A \cap B = B \cap A$ | Commutative law for intersection |
| $A \cup (B \cup C) = (A \cup B) \cup C$ | Associative law for union |
| $A \cap (B \cap C) = (A \cap B) \cap C$ | Associative law for intersection |
| $A \cup (B \cap C) = (A \cap B) \cap (A \cup C)$ | Distributive law for union |
| $A \cap (B \cap C) = (A \cap B) \cup (A \cap C)$ | Distributive law for intersection |
| $(A \cap B)^c = A^c \cap B^c$ | De Morgan law |
| $(A \cap B)^c = A^c \cup B^C$ | De Morgan law |

## EXERCISE 8.1

1. Let U = {1, 2, 3, 4, 5, 6, 7}, A = {1, 2, 3, 4}, B = {3, 4, 5}, C = {2, 3, 4, 5, 6}. Find the following:

    a. $A \cup B$                       b. $A \cap B$

    c. $A^c$                         d. $(A \cup B) \cap C$

    e. $(A \cap B) \cup C$           f. $A^c \cup B \cup C$

2. Let U denote the set of all corporations in this country and P those that made profits during the last year, D those that paid a dividend during the last year, and L those that increased their labor force during the last year. Describe the following using the three sets P, D, L, and set operations. Show the regions in a Venn diagram.

    a. Corporations in this country that had profits and also paid a dividend last year

b. Corporations in this country that either had prof its or paid a dividend last year

c. Corporations in this country that did not have profits last year

d. Corporations in this country that had profits paid a dividend, and did not increase their labor force last year

e. Corporations in this country that had profits or paid a dividend, and did not increase their labor force last year.

## 8.7. THE NUMBER OF ELEMENTS IN A SET

### Counting the Elements of a Set

This section shows the relationship between the number of elements in $A \cup B$ and the number of elements in A, B, and $A \cap B$. This is our first counting principle. The examples and exercises in this section give some applications of this. In other applications we will count the number of elements in various sets to find probability.

### The Notation $n(A)$

If A is a set with a finite number of elements, we denote the number of elements, we denote the number of elements in A by n(A).
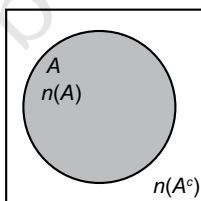


**Fig. 8.10.**

In Figure 8.10 we see the number n(A) written inside the A circle and n($A^c$) written outside the set A. This indicates that there are n(A) members in set A and n($A^c$) in set $A^c$. The number of elements in a set is also called the cardinality of the set.

There are two result that are rather apparent. First, the empty set $\phi$ has no elements n($\phi$) = 0. For the second refer to Figure 8.11 where the two sets A and B are disjoint.

### The Number, in the Union of Disjoint Sets,

If the sets A and B are disjoint, then

$$n(A \cup B) = n(A) + n(B)$$

A consequence of the last result is the following. In Figure 8.10, we are give a universal set $\cup$ and a set $A \subset U$. Then, since $A \cap A^c = \phi$ and $U = A \cup A^c$,
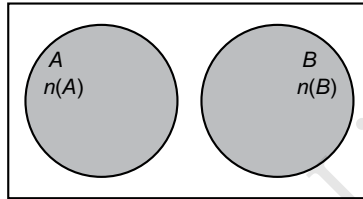
$$n(U) = n(A \cup A^c) = n(A) + n(A^c)$$



**Fig. 8.11.**

## Union Rule for Two Sets

Now consider the more general case shown in Figure 8.12. We assume that x is the number in the set A that are not in B, that is, $n(A^c \cap B^c)$. Next we have z, the, number in the set B that are not in A, $n(A^c \cap B)$. Finally, y is the number in both A and B, $n(A \cap B)$ and w is the number of elements that are neither in A nor in 8 $n(A^c \cap B^c)$. Then

$$n(A \cup B) = x + y + z$$
$$= (x + y) + (y + z) - y$$
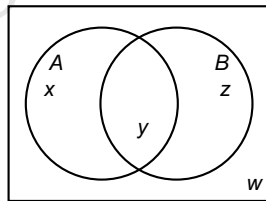$$= n(A) + n(B) - n(A \cap B)$$



**Fig. 8.12.**

Alternatively, we can see that the total $n(A) + n(B)$ counts the number in the intersection $n(A \cap B)$ twice. Thus to obtain the number in the union $n(A \cup B)$, we must subtract $n(A \cap B)$ from $n(A) + n(B)$.

### The Number in the, Union of Two Sets

For any finite Sets A and B,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

**Example 12:** *An* Application of Counting *In a survey of 120 adults, 55 said they had an egg for breakfast that morning, 40 said they had juice for breakfast and 70 said they had an egg or juice for breakfast. How many had an egg but  juice for breakfast? How many bad neither an egg nor juice for breakfast?*

**Solution:** Let U be the universal set of adults surveyed, E the set that had an egg for breakfast, and J the set that had juice for breakfast. A Venn diagram is shown in Figure 8.13a. From the survey, we have that
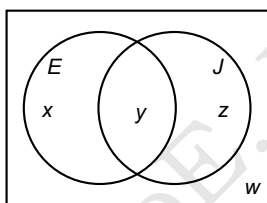
$$n(E) = 55 \qquad n(J) = 40, \qquad n(E \cup J) = 70$$



**Fig. 8.13.a**

Note that each of these is a sum. That is $n(E) = 55 = x + y$, $n(J) = 40 = y + z$ and $n(E \cup J) = 70 = x + y + z$. Since 120 people are in the universal set, $n(U) = 120 = x + y + z + w$.

The number that had an egg and juice for breakfast is given by $n(E \cap J)$ and is shown as the shaded region in Figure 8.13b. We apply the union rule:

$$\begin{aligned} n(E \cap J) &= n(E) + n(J) - n(E \cup J) \\ &= 55 + 40 - 70 \\ &= 25 \end{aligned}$$
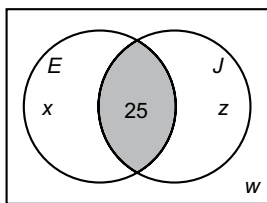


**Fig. 8.13.b**

We first place the number 25, just found, in the $E \cap J$  area in the Venn diagram in Figure 8.13b. Since the number of people who had eggs (with and without juice) is 55, then according to Figure 8.13b,

$$n(E) = 55 = x + 25$$
$$x = 30$$

Similarly, the number who had juice (with and without an egg) is 40. Using Figure 8.13b,

$$n(J) = 40 = z + 25$$
$$z = 15$$

These two results are shown in Figure 8.13c. We wish to find w = $n((E \cup J)^c)$. This is shown as the shaded region in Figure 8.13c. The unshaded region is $E \cup J$. We then have that

$$n(E \cup J) + n((E \cup J)^c) = n(U)$$
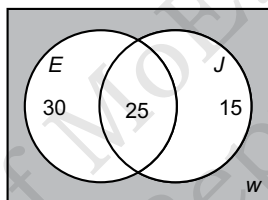$$n((E \cup J)^c) = n(U) - n(E \cup J)$$
$$w = 120 - 70$$
$$w = 50$$



**Fig. 8.1&c**

So, there were 50 people in the surveyed group that had neither an egg nor juice for breakfast

### Counting with Three Sets

Many counting problems with sets have two sets in the universal set. We will also study applications with three sets in the universal set. The union rule for three sets is studied in the extensions for this section. In the example below, deductive reasoning is used to solve for the number of elements in each region of the Venn diagram.

**Example 13:** European Travels *In a survey of 200 people that had just returned from a trip to Europe, the following information was gathered.*

- *142 visited England*
- *95 visited Ttaly*
- *65 visited Germany*
- *70 visited both England and Italy*
- *50 visited both England and Germany*

- *30 visited both Italy and Germany*
- *20 visited all three of these countries*
  a. *How many went to England but not Italy or Germany?*
  b. *How many went to exactly one of these three countries?*
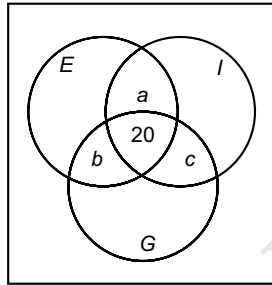  c. *How many went to none of these three countries?*



**Fig. 8.14.a**

**Solution:** Let U be the set of 200 people that were surveyed and let

E = {x/x visited England}

I = {x/x visited Italy}

G = {x/x visited Germany}

We first note that the last piece of information from the survey indicates that

$$n(E \cap I \cap G) = 20$$

Place this in the Venn diagram shown in Figure 8.14a. Recall that 70 visitel both England and Italy, that is, $n(E \cap I) = 70$. If a is the number that visitel England and Italy but not Germany, then, according to Figure 1.14a, $20 + a = n(E \cap I) = 70$. Thus a = 50. In the same way, if b is the number that visitd England and Germany but not Italy, then $20 + b = n(E \cap G) = 50$. Thus b = 30. Also if c is the number that visited Italy and Germany but not England, then $20 + c = n(G \cap I) = 30$. Thus c = 10. All of this information is then shown in Figure 8.14b.
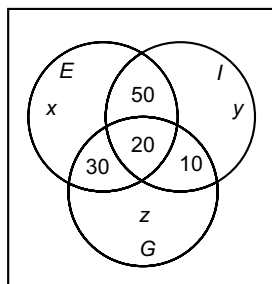


**Fig. 8.1' .b**

(a) Let x denote the number that visited England but not Italy or Germany. Then according to Figure 8.14b, 20 + 30 + 50 + x = n(E) = 142. Thus x = 42, that is, the number that visited England but not Italy or Germany is 42.

(b) Since n(I) = 95, the number that visited Italy but not England or Germany is given from Figure 8.14b by 95 – (50 + 20 + 10) = 15. Since n(G) = 65 the number that visited Germany but not England or Italy is, according to Figure 8.14b, given by 65 – (30 + 20 + 10) = 5. Thus, according to Figure 8.14c the number who visited just one of the three countries is

$$42 + 15 + 5 = 62$$

(c) There are 200 people in the U and so according to Figure 8.14c, the number that visited none of these three countries is given by

$$200 – ( 42 + 15 + 5 + 50 + 30 + 10 + 20) = 200 – 172 = 28$$
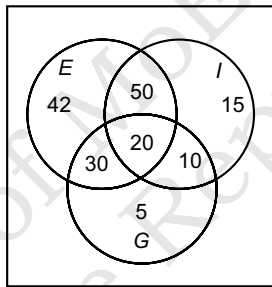


**Fig. 8.1' .c**

**Example 14:** *At the end of the day the manager of Blue Baker wanted to know how many pizzas were sold. The only information he had is listed below Use the information to determine how many pizzas were sold.*

- *3 pizzas bad mushrooms, pepperoni, and sausage*
- *7 pizzas bad pepperoni and sausage*
- *6 pizzas had mushrooms and sausage but not pepperoni*
- *15 pizzas had two or more of these toppings*
- *11 pizzas had mushrooms*
- *8 pizzas had only pepperoni*
- *24 pizzas had sausage or pepperoni*
- *17 pizzas did not have sausage*

**Fig. 8.15.a**

**Solution:** Begin by drawing a Venn diagram with a circle for pizzas that had mushrooms, a circle for pizzas that had pepperoni, and, pizzas that had sausage. In the center place three pizzas had all these toppings. See Figure 8.15a.
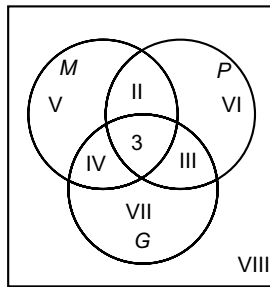
Since, 7 pizzas have pepperoni and sausage, 7 = 3 + III or III = 4. If 6 pizzas had mushrooms and sausage but not pepperoni, then IV = 6. The region for two or more of these toppings is 3 + II + Ill + IV = 15. Using III = 4 and IV = 6, that gives 3 + II + 4 + 6 = 15 or II = 2. This information is shown in Figure 8.15b.
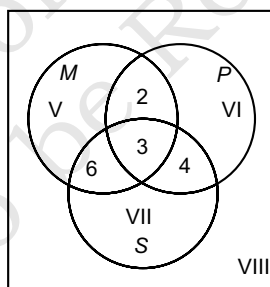


**Fig. 8.1( .b**

Given that 11 pizzas had mushrooms, V + 2 + 3 + 6 = 11 and therefore V = 0. Since 8 pizzas had only pepperoni, VI = 8. With a total of 24 pizzas in the sausage or pepperoni region and knowing that VI = 8, we have 2 + 8 + 6 + 3 + 4 + VII = 24 or VII = 1. Finally, if 17 pizzas did not have sausage then 17 = V + 2 + VI + VIII = 0 + 2 + 8 + VIII. This gives VIII = 7 and our complete diagram is shown in Figure 8.15c.

To find the total number of pizzas sold, the 8 numbers in the completed Venn diagram are added:

$$0 + 2 + 8 + 6 + 3 + 4 + 1 + 7 = 31$$
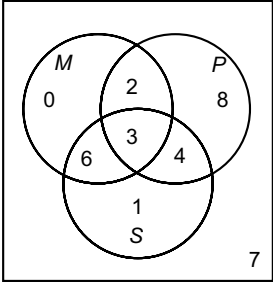


**Fig. 8.1( .c**

## 8.8. TREE DIAGRAMS

We completed a task (flipped a coin) and then completed another task (flipped the coin again). In these cases, the experiment can be diagrammed with a tree. The tree diagram is shown in Figure 8.16. We see we have a first set of branches representing the first flip of the coin. From there we flip the coin again and have a second set of branches. Then trace along each branch to find the outcomes of the experiment. If the coin is tossed a third time there will be eight outcomes.
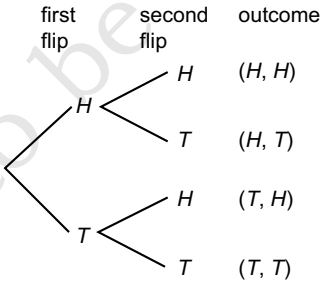


**Fig. 8.16.**

**Example 15:** *A die is rolled. If the die shows a 1 or a 6, a coin is tossed. What is the sample space for this experiment?*

**Solution:** Figure 8.17 shows the possibilities. We then have

$$S = \{(1, H), (1, T), 2, 3, 4, 5, (6, H), (6, T)\}$$

**Fig. 8.17.**

## ▌ 8.9.  CONTINUOUS SAMPLE SPACES

In all the previous examples we were able to list each outcome in the sample space, even if the list is rather long. But consider the outcomes of an experiment where the time spent running a race is measured. Depending on how the time is measured, an outcome could be 36 seconds or 36.0032 seconds. The values of the outcomes are not restricted to whole numbers and so the sample space must be described, rather than listed. In the case of the race we could 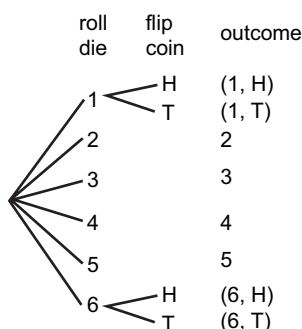say $S = \{t \mid t \geq 0,\ t \text{ in seconds}\}$. Then the event E that a person takes less than 35 seconds to run the race would be written $E = \{t \mid t < 35 \text{ seconds}\}$.

**Example 16:** *At a farmer's market there is a display of fresh oranges. The oranges are carefully weighed. What is a sample space for this experiment? Describe the event that a orange weighs 100 grams or more. Describe the event that a orange weighs between 200 and 250 grams.*

**Solution:** Since, the weight of the orange can be any positive number,

$$S = \{w \mid w > 0, \text{ win grams}\}$$

Note that $w = 0$ is not included as if the weight was zero there would be no orange! The event that an orange weighs 100 grams or more is

$$E = \{w \mid w \geq 100, \text{ win grams}\}$$

Here note that we use $\geq$, not $>$ as the value of exactly 100 grams needs to be included. The event that the orange weighs between 200 and 250 grams is

$$F = \{w \mid 200 < w < 250,\ w \text{ in grams}\}$$

where strict inequalities are used as the weight is between those values.

## EXERCISE 8.2

1. Two tetrahedrons ( 4 sided), each with equal sides numbered from 1 to 4, are identical except that one is red and the other green. If the two tetrahedrons are tossed and the number on the bottom face of each is observed, what is the sample space for this experiment?

2. Consider the sample space given in the previous exercise. Let E consist of those outcomes for which both (tetrahedron) dice show an odd number. Let F be the event that the sum of the two numbers on these dice is 5. Let G be the event that the sum of the two numbers is less than 7.

   a. List the elements of E and F.
   b. Find $E \cap F$.
   c. Find $E \cup F$.
   d. Find $G^c$.

3. A hospital carefully measures the length of ever baby born. What is a sample space for this experiment? Describe the events

   a. the baby is longer than 22 inches.
   b. the baby is 20 inches or shorter.
   c. the baby is between 19.5 and 21 inches long.

## ▌ 8.10.  SAMPLE SPACES AND EVENTS

Many people have a good idea of the basics of probability. That is, if a fair coin is flipped, you have an equal chance of a head or a tail showing. However, as were proceed to study more advanced concepts in probability we need some format definitions that will both agree with our intuitive understanding of probability and allow us to go deeper into topics such as conditional probability. This will tie closely to work we have done learning about sets.

### The Language of Probability

We begin the preliminaries by stating some definitions. It is very important to have a clear and precise language to discuss probability so pay close attention to the exact meanings of the tenns below.

### Experiments and Outcomes

An experiment is an activity that has observable results.

An outcome is the result of the experiment.

The following are some examples of experiments. Flip a coin and observe whether it falls "heads" or "tails." Throw a die (a small cube marked on each face with from one to six dots 1) and observe the number of dots on the top face. Select a transistor from a bin and observe whether or not it is defective.

The following are some additional terns that are needed.

### Sample Spaces and Trials

A sample space of an experiment is the set of all possible outcomes of the experiment. Each repetition of an experiment is called a **trial**.

For the experiment of throwing a die and observing the number of dots on the top face the sample space is the set

$$S = \{1, 2, 3, 4, 5, 6\}$$

In the experiment of flipping a coin and observing whether it falls heads or tails, the sample space is S = {heads, tails} or simply S = {H, T}.

**Example 17:** *Determining the Sample Space An experiment consists of noting whether the price of the stock of the Ford Corporation rose, fell, or remained unchanged on the most recent day of trading. What is the sample space for this experiment?*

**Solution:** There are three possible outcomes depending on whether the price rose, fell, or remained unchanged. Thus the sample space S is

$$S = \{\text{rose, fell, unchanged}$$

**Example 18:** *Determining the Sample Space Two dice, identical except that one is green and the other is red, are tossed and the number of dots on the top face of each is observed. What is the sample space for this experiment?*

**Solution:** Each die can take on its six different values with the other die also taking on all of its six different values. We can express the outcomes as order pairs. For example, (2, 3) will mean 2 dots on the top face of the green die and 3 dots on the top face of the red die. The sample space S is below. A more colourful version is shown in Figure 8.18.

$$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),$$
$$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),$$
$$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),$$
$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),$$

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

(1, 1) (1, 2) (1, 2) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

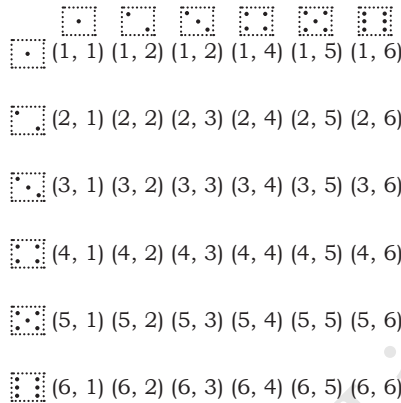(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

**Fig. 8.18.**

## 8.11. CONTINGENCY

A contingency table displays frequencies for combinations of two categorical variables. Analysts also refer to contingency tables as crosstabulation and two-way tables.

Contingency tables classify outcomes for one variable in rows and the other in columns. The values at the row and column intersections are frequencies for each unique combination of the two variables.

Use contingency tables to understand the relationship between categorical variables. For example, is there a relationship between gender (male/female) and type of computer (Mac/PC).

I love these tables because they organize your data and allow you to answer diverse questions, this post, learn about contingency tables, including how to interpret, graph, and analyze them.

### Example: Contingency Table

The contingency table example below displays computer sales at our fictional store. Specifically it describes sales frequencies by the customer's gender and the type of computer purchased. It a two-way table (2 × 2). I cover the naming conventions at the end.

|          | PC | Mac | Row Total |
|----------|----|-----|-----------|
| **Male** | 66 | 40  | 106       |

| Female | 30 | 87 | 117 |
| Column Totals | 96 | 127 | 223 |

In this contingency table, columns represent computer types and rows represent genders. Cell values are frequencies for each combination of gender and computer type. Totals are in the margins. Notice the grand total in the bottom-right margin.

At a glance, it's easy to see how two-way tables both organize your data and paint a picture or t results. You can easily see the frequencies for all possible subset combinations along with total.

## Marginal and Conditional Distributions in Contingency Table

Contingency tables are a fantastic way of finding marginal and conditional distributions. These two distributions are types of frequency distributions. Learn more about Frequency Tables: How to Make and Interpret.

## Marginal Distribution

These distributions represent the frequency distribution of one categorical variable without regard for other variables. Unsurprisingly, you can find these distributions in the margins of a contingency table.

The following marginal distribution examples correspond to the blue highlights.

|  | PC | Mac | Row Totals |
| --- | --- | --- | --- |
| Male | 66 | 40 | 106 |
| Female | 30 | 87 | 117 |
| Column Totals | 96 | 127 | 223 |

**Example 19:** The marginal distribution of gender without considering computer type is the following:

- **Males:** 106
- **Females:** 117

Alternatively, the marginal distribution of computer types is the following:

- **PC:** 96
- **Mac:** 127

Learn more about Marginal Distributions

## Conditional Distribution

For these distributions, you specify the value for one of the variables in the contingency table a then assess the distribution of frequencies for the other variable. In other words, you conditior the frequency distribution for one variable by setting a value of the other variable. That might sound complicated, but it's easy using a contingency table. Just look across one row or down or column.

|              | PC  | Mac | Row Totals |
|--------------|-----|-----|------------|
| Male         | 66  | 40  | 106        |
| Female       | 30  | 87  | 117        |
| Column Totals | 96 | 127 | 223        |

For example, the conditional distribution of computer type for females is the following:

- **PC:** 30
- **Mac:** 87

Alternatively, the conditional distribution of gender for Macs is the following:

- **Males:** 30
- **Females:** 87

Learn more about Conditional Distributions.

## Finding Relationships in a Contingency Table

In the contingency table below, the two categorical variables are gender and ice cream flavour preference. This is a two-way table ($2 \times 3$) where each cell represents the number of times male and females prefer a particular ice cream flavour. The CSV datasheet shows one format you can use to enter the data into your software: Flavour Preference,

| Gender | Chocolate | Strawberry | Vanilla | Total |
|--------|-----------|------------|---------|-------|
| **Female** | 37    | 17         | 12      | 66    |
| **Male**   | 21    | 18         | 32      | 71    |
| **Total**  | 58    | 35         | 44      | 137   |

How do we go about identifying a relationship between gender and flavour preference?

If there is a relationship between ice-cream preference and gender, we'd expect the condition a distribution of flavours in the two gender rows to differ. From the contingency table, females are more likely to prefer chocolate (37 vs. 21), while males prefer vanilla (32 vs. 12). Both genders have an equal preference for strawberry. Overall, the two-way table suggests that males and x.

The Total column indicates the researchers surveyed 66 females and 71 males. Because we have roughly equal numbers, we can compare the raw counts directly. However, when you have unequal groups, use percentages to compare them.

## Row and Column Percentages in Contingency Tables

Row and column percentages help you draw conclusions when you have unequal numbers in the margins. In the contingency table example above, more women than men prefer chocolate, but how do we know that's not due to the sample having more women? Use percentages to adjust for unequal group sizes. Percentages are relative frequencies. Learn more about Relative Frequencies and their Distributions.

Here's how to calculate row and column percentages in a two-way table.

- **Row Percentage:** Take a cell value and divide by the cell's row total.
- **Column Percentage:** Take a cell value and divide by the cell's column total.

For example, the row percentage of females who prefer chocolate is simply the number of observations in the Female/Chocolate cell divided by the row total for women: $37/66 = 56\%$.

The column percentage for the same cell is the frequency of the Female/Chocolate cell divided the column total for chocolate: $37/58 = 63.8\%$.

## Interpreting Percentages in a Contingency Table

The contingency tab le below uses the same raw data as the previous table and displays both row and column percentages. Note how the row percentages sum to 100% in the right margin while the column percentages sum to 100% at the bottom.
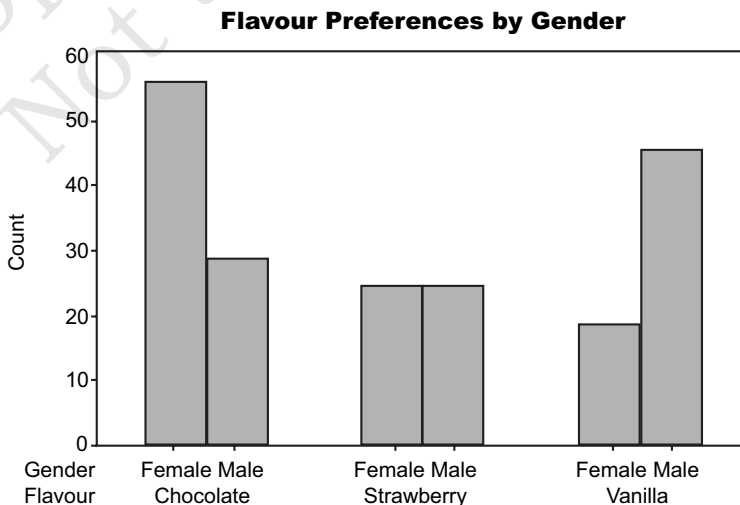
| Gender | Chocolate | Strawberry | Vanilla | Row Total |
|--------|-----------|------------|---------|-----------|
| **Female** | Raw: 37<br>Row%: 56%<br>Col%: 63.8% | Raw: 17<br>Row%: 25.8%<br>Col%: 48.6% | Raw: 12<br>Row%: 18.2%<br>Col%: 28.8% | Raw: 66<br>Row%: 100% |
| **Male** | Raw: 21<br>Row%:29.6%<br>Col%: 36.2% | Raw: 18<br>Row%: 25.4%<br>Col%: 51.4% | Raw: 32<br>Row%: 45.0%<br>Col%: 71.2% | Raw: 71<br>Row%: 100% |

Whether you focus on row percentages or column percentages in a contingency table depends on the question you're answering. In our case, we want to know whether flavour preference depends on gender. Because the two genders display in separate rows, we'll look for difference in the row percentages.

56% of females prefer chocolate versus only 29.6% of males. Conversely, 45% of males prefer vanilla, while only 18.2% of females prefer it. These results reconfirm our previous findings usir the raw counts.

## How to Graph a Contingency Table

You can use bar charts to display a contingency table. The following clustered bar chart shows the row percentages for the previous two-way table. I've set the graph to cluster the female and male pairs of bars together for each flavour, making comparisons easier. I think it gives a nice oomph to the tabular results.



**Flavour Preferences by Gender**

This bar chart reiterates our conclusions from the contingency table. Women in this sample prefer chocolate, men favour vanilla, and both genders have an equal preference for strawberry,

We've already looked at various ways to analyze a contingency table. Here are two more methods that take it to another level.

Contingency tables are a fantastic way to display and find various types of probabilities. Use these tables to calculate joint, marginal, and conditional probabilities. I've written an article about calculating probabilities using two-way tables, and it includes all the definitions, notation and formulas you need. Read about Using Contingency Tables to Calculate Probabilities.

In this post, we looked for a relationship between gender and ice cream preference by noting they differences between counts and row percentages in the contingency table. If we're using this sample to draw inferences about the entire population of ice cream consumers, we'll need to u a hypothesis test to evaluate the relationship.

In other words, are the differences we noticed in the sample large enough to support the notion that a relationship exists in the population? Or can we chalk up the differences to random sampling error? Learn how the chi-square test of independence can help us out by analyzing contingency tables!

## Naming Conventions for Contingency Tables

Contingency tables come in a variety of flavours. The key considerations for naming the types an the number of categorical variables and the number of values for each categorical variable.

## Number of Categorical Variables

You must have at least two categorical variables to create a contingency table. When you have two variables, it's a two-way table. If you have three, it's a three-way table, and so on.

For example, suppose we run a computer store and record the sales using the two categorical variables of gender and computer type. Those variables create a two-way contingency table. If add a third categorical variable for store location, it becomes a three-way table.

How do you present a three~way table?

Because contingency tables display in two dimensions, you need multiple tables to represent anything more than a two-way table.

## Number of Rows and Columns

The rows represent values of one categorical variable, while the columns denote the values of another. Analysts indicate the number of values for each variable by describing these tables as an A × B contingency table, where A represents the number of rows and B signifies the number of columns.

For example, at our computer store, Gender has two possible values and computer type has two values (PC and Mac). Hence, we have a 2 × 2 contingency table. If we add a third type of computer as a new column, it becomes a 2 × 3 table.

### EXERCISE 8.3

1. A bag contains 50 identical balls, 20 of picking are green and the rest red. Find the probability of picking at random a red ball from the bag.

2. A bag contains 36 identical marbles. 10 are red, 12 are green and the rest yellow, Find the probability of selecting . at random a yellow marble from the bag.

3. A number is selected at random from the set

   S = {1,2,3, .. . ,15}.

   What is the probability that the number is even?

4. There are 100 cars in a car park, 28 of them are blue and 34 red. If a car is selected at random from the car, what is the probability that it is neither blue nor red?

5. A fair coin is tossed twice. Write down the set of possible outcomes.

   What is the probability of obtaining?

   (i) exactly 2 heads;            (ii) no head;

   (iii) at least one head;        (iv) a head and a tail.

6. Find the probability that a student selected at random from the class is

   (i) 14 years old

   (ii) less than 14 years old

   (iii) greater than 16 years old

   (iv) between 13 and 17 years old.

+. The probability that an event A occurs is $\frac{1}{5}$ and the probability that an event B occurs is $\frac{1}{3}$. Assuming that the two events are independent events, find the ·probability that

    (i) A and B both occur;        (ii) Neither event occurs;

   (iii) At least one of the events occurs.

,. In a contest, Daniel, Henry and Albert are asked to solve a problem, the respective probabilities that they solve the problem are $\frac{1}{6}, \frac{1}{8}$ and $\frac{1}{3}$. Calculate the probability that:

   (a) none of them solves the problem;

   (b) at least one of them solves the problem;

   (c) only one of them solves the problem

-. The probability that Micheal goes to school on a market day is $\frac{1}{5}$ and the probability that his brother also attends school the same day is $\frac{3}{4}$. What is the probability that

   (i) both were absent?

  (ii) at least one was present?

  (iii) exactly one of them is present?

%&. Mrs. Emino is expecting a baby. The probability that the baby will be a boy is $\frac{1}{2}$ and the probability that the baby will have blue eyes is $\frac{1}{4}$. What is the probability that she will have a blue-eyed baby?

%%. A bag contains 10 balls that differ only in colour; 4 are blue and 6 are red. Two balls are plcked, one after the other, with replacement. What is the probability that:

   (i) both are red

  (ii) both are of the same colour.

%&. A box contains 5 white balls, 2 black balls and 3 red balls of the same size. A ball is selected at random from the box and then replaced. A second ball is then selected. Find the probability of obtaining

   (a) two red balls

   (b)  two white balls or two black balls

   (c)  one black ball and one red ball in any other.

**%.**  The probabilities that two boys pass an examination are $\dfrac{2}{3}$ and $\dfrac{5}{8}$. Find the probability that

   (i)  the two boys pass the examination.

   (ii)  only one of the boys passes the examination.

**%.**  The probabilities of two candidates, A and B, passing an examination are $\dfrac{2}{3}$ and $\dfrac{3}{4}$, respectively. Find the probability that:

 (a)  **only** one candidate will pass;

 (b)  **at least** one candidate will pass.